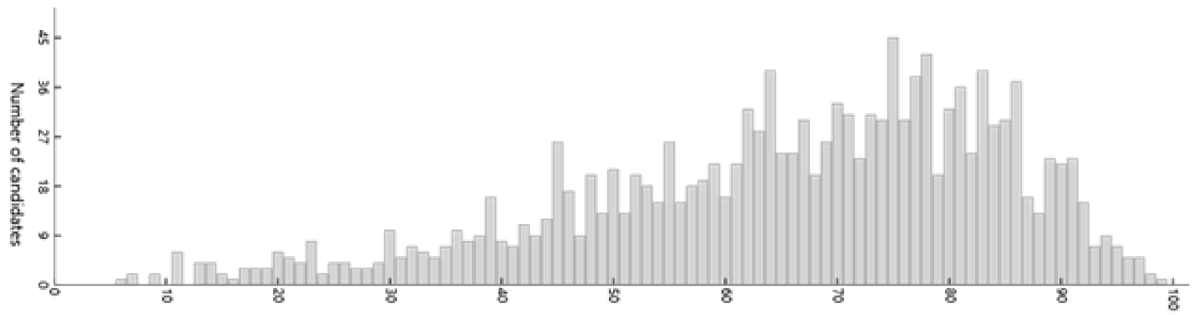
Government of **Western Australia**
School Curriculum and Standards Authority

2019 ATAR course examination report:
Mathematics Specialist

Year	Number who sat	Number of absentees
2019	1435	32
2018	1546	21
2017	1463	12
2016	1427	17

Examination score distribution–Written



Summary
The examination consisted of Section One: Calculator-free and Section Two: Calculator-assumed.

Attempted by 1435 candidates	Mean 65.34%	Max 99.25%	Min 6.08%

Section means were:

Section One: Calculator-free	Mean 68.54%		
Attempted by 1435 candidates	Mean 23.99(/35)	Max 35.00	Min 0.74
Section Two: Calculator-assumed	Mean 63.61%		
Attempted by 1435 candidates	Mean 41.35(/65)	Max 64.62	Min 0.76

General comments
The paper appeared to be accessible for this Mathematics Specialist cohort. The paper contained a range of questions allowing the typical Specialist candidate to show facility with key standard concepts. There were still elements in each question to discriminate amongst candidates. The questions requiring concept development/proof (such as Questions 9 and 19) allowed the most capable candidates to excel.

The length of the paper was appropriate, as evident by the high percentage of candidates attempting the final questions. Marks were relatively well spread and similar to the distribution in 2018. There appeared to be a significant number of candidates who were not able to answer straightforward questions, or indeed offer any response to questions.

A good number of more capable candidates were able to show elegant and original solutions. This was evident particularly for Question 7 and obviously in Question 9 and 19 where the correct use of mathematics was well rewarded.

Advice for candidates

- Know key exact trigonometric values.
- Improve the command of essential algebraic processes.
- Write a clear conclusion to each question.
- Improve the legibility of digits.

Advice for teachers

- Give students opportunities for proving mathematics results. The key distinction is separating out given information from what has to be proved.
- Improve students conceptual understanding of vectors, specifically those in 3D.
- Encourage students to discover the range of a function by graphing the function, in many cases from knowing the sketch of the reciprocal of the required function.
- Clarify the sketching of a solution curve in instances where the slope field value is undefined at a point.
- Insist on students using correct mathematics notation and vocabulary. References to 'it' is not helpful in answering questions that explain aspects of mathematics. Comments such as the 'standard deviation decreases' are vague when it was required to state that the 'standard deviation of the sample mean' decreases as the sample size increases.

Comments on specific sections and questions

Section One: Calculator-free (47 Marks)

Candidates performed well with many straightforward questions in this section.

Question 1 attempted by 1425 candidates Mean 3.42(/4) Max 4 Min 0
Most candidates scored reasonably well. Errors were mainly confined to not knowing exact trigonometric values, or not being able to work with fractions.

Question 2 attempted by 1422 candidates Mean 5.12(/6) Max 6 Min 0
In part (a), most candidates knew that they needed to show how they obtained zero by showing the value of each term when $z = 2i$ was substituted. In part (b), the majority knew that the conjugate factor $(z + 2i)$ was also a factor and hence $z^2 + 4$ was a factor.

Question 3 attempted by 1411 candidates Mean 3.84(/5) Max 5 Min 0
In part (a), candidates who decided to effectively obtain the LCD as $x^3(x+3)$, ran into trouble with the numerators. Poor algebra was at the heart of difficulties here. Most candidates recognised that the natural logarithm of an absolute value was required and knew that a constant of integration was required for part (b). Candidates had trouble integrating the term $2x^{-2}$ with many invoking a natural logarithm.

Question 4 attempted by 1435 candidates Mean 4.02(/7) Max 7 Min 0
Part (a) was straightforward and done well. In part (b), the statement of the domain was done well. Part (c) was more difficult and was a discriminating question of the paper. Candidates needed to sketch the graph of $y = \sqrt{x} - 1$ and then considered sketching the reciprocal graph $y = \dfrac{1}{\sqrt{x} - 1}$. This would have made the statement of the range of the required function accessible to candidates. For part (d), many candidates were confused, trying to justify why the statement was false. Contradictory reasons were given. Vague reasons along the lines of 'false since they have different domains and ranges', were not accepted. If a candidate stated the correct (and different) domains and ranges then that was accepted.

Question 5 attempted by 1431 candidates Mean 4.60(/6) Max 6 Min 0
The sketch of the inverse function was done well. Accuracy was required for full marks in part (a). Specific ordered pairs in the graph of $y = g(x)$ had to be sought to then know which ordered pairs to plot for $y = g^{-1}(x)$. In part (b), poor algebra skills stopped candidates from getting close to the correct defining rule. The issue in considering the positive/negative of a square root was not demonstrated by the majority of the cohort.

Question 6 attempted by 1423 candidates Mean 4.16(/6) Max 6 Min 0
This question tested evaluating an integral using a given substitution. Most candidates knew what had to be done, but poor algebra in simplifying the integrand gave incorrect answers. These errors lead to a greatly simplified integrand not requiring a trigonometric identity, and could not be awarded full marks.

Question 7 attempted by 1380 candidates Mean 2.54(/4) Max 4 Min 0
Most candidates scored three out of four marks. Only the most capable candidates could demonstrate sound understanding of what constituted a $1-1$ function. Many declared that f had to be $1-1$, but then drew a function that was clearly not $1-1$, indicating that they were repeating an idea taught but did not understand it.

Question 8 attempted by 1420 candidates Mean 4.28(/5) Max 5 Min 0
In part (a), most candidates realised that they needed to obtain the x coordinate as the subject from the implicitly defined equation but algebraic skills were lacking. Part (b) was done well given that the expression for the volume was provided. Errors were mainly arithmetic slips and/or not being able to read their own work ($1+y^3$ often morphed into $1+y^2$ and the factor of π was often omitted).

Question 9 attempted by 1259 candidates Mean 0.64(/4) Max 4 Min 0
Candidates were not able to nominate the last root z_{n-1} as $cis\left(\frac{2(n-1)\pi}{n}\right)$ where $k = n-1$.
This showed that many candidates did not understand how the roots are formed. Those that obtained the expression $cis((n-1)\pi)$ then needed to consider the cases where n was even/odd. A small number of candidates knew the result from prior exposure to this idea, but that they could not develop the detail to arrive at their result. High performing candidates scored well here.

Section Two: Calculator-assumed (85 Marks)
The Calculator-assumed section enabled candidates to recall many well known concepts for routine type questions. The poor performance in Question 19 highlighted the need for candidates to have had greater exposure to the demands of a proof.

Question 10 attempted by 1371 candidates Mean 2.66(/5) Max 5 Min 0
Many candidates chose to skip this question. Both parts of the question emphasized conceptual understanding. In part (a), many candidates stated either $z_0 = 2,\ z_0 = -2,\ z_0 = 2i$ rather than $z_0 = -2i.$ The key idea was to examine the locus and verbalise the idea 'the argument of z FROM z_0 is equal to $\frac{\pi}{4}$'. Similarly in part (b), the idea was to verbalise the question as 'the distance of z FROM i is to be a minimum'. Many drew the distance on the diagram indicating they understood how the minimum was obtained. Errors were introduced in reading the scale of the Argand diagram or not being able to work with exact values (despite having a CAS calculator).

Question 11 attempted by 1428 candidates Mean 5.17(/6) Max 6 Min 0
The use of the differential equation was done well in part (a). Drawing the solution curve proved problematic for many in part (b). Most drew the curve showing a vertical orientation at $y = -1$, but then did not draw a curve that adequately followed the slope field indicators. A small group of candidates opted to only draw the solution curve for $y \geq -1$ (i.e. only drew the top portion of the curve), despite then correctly obtaining the defining rule for the entire solution curve in part (c). Part (c) was well done, including the determination of the constant of integration.

Question 12 attempted by 1430 candidates Mean 7.98(/10) Max 10 Min 0
Part (a) was done well, yet with the use of a CAS calculator it is difficult to know how this can be mismanaged. Candidates performed well in part (b). Many opted to read the coordinates directly from the diagram, correctly stating that $\mathrm{Re}(z) = -2$ but incorrectly stating that $\mathrm{Im}(z) = 3.5$. Part (c) was straightforward, yet candidates were often inconsistent in using what they had answered in parts (a) and (b). The main issue in part (d) was the apparent difficulty in interpreting the scale divisions to plot the arguments correctly. It appeared some candidates were not familiar with this task. In part (e), candidates should be instructed to use vocabulary that emphasises the geometric transformations. Language such as 'the angle decreases by $\frac{\pi}{4}$ was not the required language in the context of geometric transformations.

Question 13 attempted by 1421 candidates Mean 5.87(/10) Max 10 Min 0
Performance was good in part (a), with most able to indicate the starting point on the path. With part (b), most candidates were able to differentiate the vector function for displacement and then evaluate to determine the initial velocity vector. A small minority were able to draw this vector on the diagram correctly, with most drawing the vector that was either too small, following a direction along the curve or showing a particular ordered pair (as a point). Forming the expression for the distance travelled was done well in part (c). Many candidates did not provide correct notation. The task of writing the cartesian equation in part (d) was a more challenging question. Algebraic errors impeded many candidates. Those using a square root to obtain y as the subject had to include both the positive and negative values to obtain the full path, which clearly shows values above and below $y = 1$.

Question 14 attempted by 1420 candidates Mean 8.02(/10) Max 10 Min 0
Part (a) was done well, with many stating or inferring that the distribution of the sample mean will be normal. Candidates need to be careful with their declaration of the standard deviation of the sample mean to distinguish it from the variance of the sample mean. Part (b) was a straightforward question with the use of a CAS calculator. Explanations in part (c) were of a reasonable standard. Candidates should have referred to the standard deviation of the sample mean in their reasoning. Part (d) was reasonably routine for this level and done well. Candidates could not simply give an answer from their CAS calculator for full marks. Most candidates rounded their answer up to the nearest integer value.

Question 15 attempted by 1421 candidates Mean 6.11(/9) Max 9 Min 0
Part (a) was a routine question and candidates performed well. Candidates that indicated the critical z score and showed how the interval was formed could score full marks. Not knowing the value of the population standard deviation caused confusion for many. In Q15 (b), those that wrote $\frac{\sigma}{\sqrt{n}} = 3$ were invariably successful. This question required the correct decimal value be given to the prescribed number of decimal places. Part (c) was answered well. Candidates were well prepared, and a good number were prepared to say that it could not be determined due to the inherent nature of random sampling.

Question 16 attempted by 1394 candidates Mean 8.10(/12) Max 12 Min 0
3D vectors continues to be a weakness for most candidates. In part (a), candidates seemed unprepared for this question and simply converted the cartesian equation into a vector equation thinking that this was answering the question. Markers could not infer what the normal vector was when a candidate wrote the vector equation. Part (b) was a straightforward question asking for the intersection between a line and a plane. Many candidates presented extremely poor notation. Part (c) was not well understood at all except for the best candidates. There was a lack of explanation as to what candidates were attempting to do or which vectors they were using. Those that cleverly opted to use the vector form for the equation $\underset{\sim}{r} = \underset{\sim}{p} + \lambda \underset{\sim}{d}_1 + \mu \underset{\sim}{d}_2$ had the easier task. For part (d), many knew what they needed to do, but were let down by inaccuracies or by not correctly stating a clear conclusion as to whether the line was a tangent to the sphere.

Question 17 attempted by 1410 candidates Mean 4.87(/8) Max 8 Min 0
In part (a), many did not know how to write the differential equation that represented 10% instantaneous growth. Part (b) was not done well. It appeared that candidates were not sufficiently agile in being able to change the form of the given differential equation to determine the correct values for a, b and $c.$ Many decided that they would use the separation of variables approach (as if they did not have a CAS calculator). Follow-through for part (c) was applied in the marking, but candidates had to provide the calendar year in which the population would have doubled. For part (d), most understood that a comment was required about the contrast in the variation in population between the models. Comments along the lines of ‘part (a) is exponential and part (b) is logistic’ did not answer the question.

Question 18 attempted by 1391 candidates Mean 5.55(/11) Max 11 Min 0
Part (a) was done well by most candidates. Part (b) was also answered well, but candidates could most easily show this by solving for α after stating that $y(0) = -80$. The use of the related rates concept was well known in part (c), but candidates were confusioned as to what θ represented and therefore what value needed to be used. Many simply thought the question wanted $\frac{dy}{d\theta}$ and so did not factor in the value for $\frac{d\theta}{dt} = \frac{\pi}{36}$. Generally, values were given correctly to 0.01 metres per second. In part (d), the question clearly stated that $y(t)$ needed to be shown as satisfying simple harmonic motion. The majority went ahead and examined $y(\theta)$ which was the easier task. The better candidates scored well here. Quite a good number knew what part (e) required, but due to the confusion as to the value for θ represented, very few obtained the correct answer. The ferris wheel drawn in the question provided a hint as to which instant in time would provide the condition for equal speeds.

Question 19 attempted by 1290 candidates Mean 0.68(/4) Max 4 Min 0
The notion of proving a result was a major candidate weakness. Many were content to quote or apply the formula for the distance from a point to a plane. This clearly is not proof, and as a result many scored zero marks for this type of response. A very small minority of candidates displayed the requisite ideas to develop the result from first principles, scoring full marks.